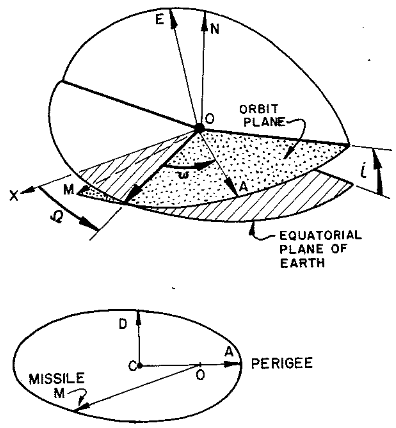
ANGLE-DAMPED DOPPLER TRACKING SYSTEM

by R. LIEBER and S. SHUCKER
Missile and Surface Radar Division
DEP, Moorestown, N. J.

This simple ground tracking system is capable of providing highly accurate orbital data in nearly real time. The system tracks orbital or ballistic bodies obeying well-defined laws. The sensor outputs are two angles (or equivalently direction cosines) and rate of change of line-of-sight distance. The data is combined in a computer to give present and future positions of the body. The system is useful for all tracking operations except for powered flight.

Fig. 1—Orbital parameters; a = semimajor axis = AC; e = eccentricity = $[1 - (DC/AC)^2]^{1/2}$; i = inclination angle; ω = argument of perigee; Ω = right ascension of ascending node; τ = time of perigee passage; O = center of earth; OX = direction of vernal equinox.



WITH THE ADVENT of ballistic missiles, satellites, and space vehicles, the tracking problem has changed.

Formerly, the total flight of tracked bodies was powered and under a form of guidance that allowed the tracking station no real *a priori* knowledge of the flight path. Vehicle maneuverability was high and position determination had to be instantaneous; wideband systems were required. Because of motive power limitations in the tracked body, tracking distances were not extreme.

Now, in the missile and space era, the percentage of powered flight to non-powered flight has decreased radically. In the case of a ballistic missile, the powered flight may be only 20 percent of the total flight. With satellite and space vehicles, the powered flight time is insignificant. Yet, the requirement for *tracking distance* has become tremendous—ranges are measured in tens of thousands of miles, as compared to the tens of miles in the old era.

TRACKING PROBLEM SOLUTIONS

Fortunately the two current problems can be compatible. The unpowered flight allows for an analytic problem formulation leading to the use of only narrow-bandwidth information, which produces the tracking distance through the use of compatible receivers and space-vehicle-mounted transmitters. Because of their inherent simplicity (again due to narrow information bandwidth) the space-vehicle-mounted transmitters have acceptable (to space-vehicle designers) weight and power-drain characteristics.

The angle-damped doppler system:

1) takes advantage of *a priori* information to formulate the tracking problem such that all narrow-bandwidth information can be best used;
2) gathers all available beacon-transmitted narrow-bandwidth information for use in the solution to the tracking problem;
3) presents a problem solution, using all of the gathered information, in nearly real time.

COMPUTER EQUATIONS

The outputs of the sensor portion of the angle damped doppler system are range rate $\dot{r}$, azimuth A, and elevation E. These quantities form the computer input.

The sensor outputs are written as functions of the orbital parameters (Fig. 1):

$$\begin{aligned} \dot{r} &= G\ (a, e, i, \omega, \Omega, \tau) \\ A &= H\ (a, e, i, \omega, \Omega, \tau) \\ E &= I\ (a, e, i, \omega, \Omega, \tau) \end{aligned} \tag{1}$$

Equations 1 are linearized by expanding them in Taylor's series form and retaining only first order terms:

$$\begin{aligned} \Delta\dot{r} &= \frac{\partial G}{\partial a}\,\Delta a + \ldots + \frac{\partial G}{\partial \tau}\,\Delta\tau \\ \Delta A &= \frac{\partial H}{\partial a}\,\Delta a + \ldots + \frac{\partial H}{\partial \tau}\,\Delta\tau \\ \Delta E &= \frac{\partial I}{\partial a}\,\Delta a + \ldots + \frac{\partial I}{\partial \tau}\,\Delta\tau \end{aligned} \tag{2}$$

Solutions for $\Delta a, \ldots \Delta\tau$ are found by minimizing the weighted sums of squares of the differences between the left and right hand sides of equations 1 and 2. This leads to the normal equations:

$$\begin{bmatrix} A_{11} & \cdots & A_{16} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ A_{61} & \cdots & A_{66} \end{bmatrix} \begin{bmatrix} \Delta a \\ \Delta e \\ \Delta i \\ \Delta\omega \\ \Delta\Omega \\ \Delta\tau \end{bmatrix} = \begin{bmatrix} B_1 \\ \circ \\ \circ \\ \circ \\ \circ \\ B_6 \end{bmatrix} \tag{3}$$

Where: B_1 to B_6 contain the observed values of $\dot{r}$, A, E. The computer solves these equations, as follows:

1) An initial estimate of the orbital elements $a \ldots \tau$ is made.
2) From the initial estimates computed vales, r_c, A_c and E_c are determined (equations 1).
3) At each measurement instant, the terms $\Delta r = r_m - r_c$, etc. are formed (equations 2). Subscript m denotes measured values.
4) The values of the coefficients $A_{11} \cdots$ and $B_1 \cdots$ are computed.
5) The equations 3 are solved for $\Delta a \cdots \Delta\tau$, the corrections to initial estimates of orbital elements.
6) The procedure is repeated until the $\Delta a \cdots \Delta\tau$ corrections are below some desired value. During the repetitive procedure, the angle data is given progressively less weight, finally being used only to damp the computation. The name, *angle-damped doppler system*, derives from this procedure.

Observation of the results of extensive digital simulation show that the computation system is analogous to a linear servo. Drawing on the linear-servo theory, the computation system and dual properties are shown in Fig. 2.

MEASUREMENT SUBSYSTEM

The specification of the measurement subsystem can be carried through many levels of technical detail. One usually starts with a final accuracy specification from the customer and proceeds from that point to define the allowable output errors in the measurement device. Further specification of the system to meet tracking distance and dynamics requirements leads to choice of beacon power, receiver sensitivity, antenna gains and tracking loop configurations. The following deals only with the customer accuracy requirements and the steps taken to translate them into allowable measurement-system output errors.

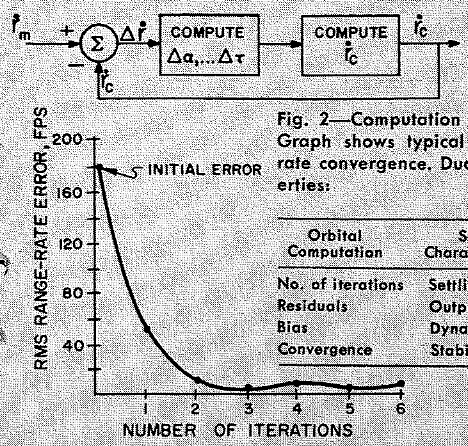


Fig. 2—Computation system. Graph shows typical range-rate convergence. Dual Properties:

Orbital Computation	Servo Characteristics
No. of iterations	Settling time
Residuals	Output noise
Bias	Dynamic lag
Convergence	Stability

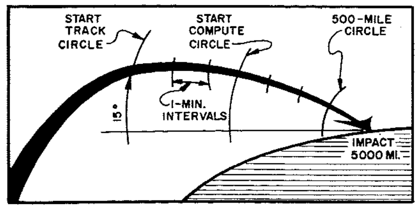


Fig. 3—Target geometry.

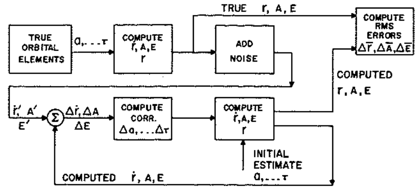


Fig. 4—System simulation.

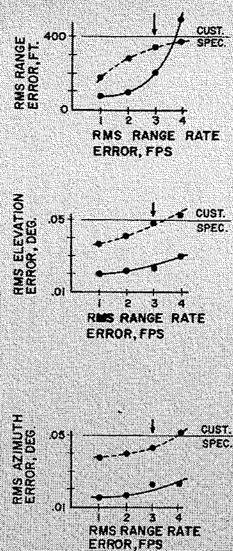


Fig. 5—Simulation results. RMS angular measurement errors: dashed curves, 0.25°; solid curves, 0.1°.

Customer Specification

The customer specification for a given missile tracking application is:

1) Data-taking span limit to be chosen by contractor.
2) System rms error over the data taking span: range $\leq$ 400 feet, azimuth $\leq 0.05°$, and elevation $\leq 0.05°$.
3) Output in real time starting at 500-mile slant ranges measured from impact point.
4) Site within 25 miles of impact.
5) ICBM trajectory as given in Fig. 3.
6) Frequency, 215 to 265 Mc.

Derivation of Allowable Data Span

Critical to meeting the customer output requirements is the allowable data-taking span, since the computation is a curve-fitting process; hence, the more data, the better the curve fit. The first part of the investigation is then concerned with the trajectory of the body to be tracked. From Fig. 3, a knowledge of the computation time requirements, and the 500-mile slant-range specification, the lower limit of the data-taking time span can be pinned down. The computation time has previously been determined to be about 3 minutes for thirty data points and 90-percent computation convergence to final values. The 500-mile and three-minute computation time is noted on Fig. 3. To determine the upper limit for data taking, the effects of propagation as related to tracking elevation angles must be considered. A conservative lower limit on the tracking elevation angle from a propagation standpoint is 15°. That is, for a reasonable choice of tracking device, the accuracy of the device will not be changed to a first order by refraction and multipath at elevation angles greater than 15°. The 15° angle is plotted on Fig. 3 and its intersection with the trajectory gives the starting point of the data-taking interval. This interval is then found to be about 3 minutes in extent. To allow for a safety factor in time, a 150-second data span is chosen for simulation purposes, giving a 30-second margin in the design of the real system. This margin can be used for search, tracking, or computation.

Tracking Subsystem Output Requirements

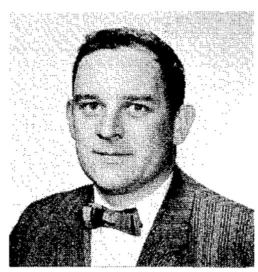
To determine the tracking subsystem output requirements, a band of values for angle and range-rate accuracy is chosen and used in a digital simulation. The simulation (Fig. 4) is carried out in the following steps:

1) A set of orbital parameters is fitted to the customer-specified missile trajectory (Fig. 3).
2) The orbital parameters are converted to values of range rate, azimuth and elevation as seen at the impact point.
3) Noise is added to the values of step two.
4) A set of initial values of orbital elements are chosen that correspond to missile impact approximately 100 miles from the true impact.
5) The noise values of step three, plus the estimated initial elements of step four, are used in the orbital calculation to see how closely the set of true orbital elements of step one can be recovered.
6) The process is carried for three cycles of iterations and the rms range, azimuth, and elevation errors between the computed trajectory and the trajectory of step two are computed (Fig. 5).
7) The curves of step six are used to define the output requirements.

From the curves of Fig. 5 and the customer's specification, the tracking subsystem output requirements are seen to be: range-rate accuracy, ≤ 3 fps; and angle accuracy, $\leq 0.25°$.

CONCLUSIONS

Excepting the realization of the range rate accuracy, which requires some special attention due to the customer's choice of frequency, the tracking subsystem requirements are modest and lead to very simple instrumentation. Thus, by proper use of *a priori* information and the proper combination of available measurements, quite respectable tracking accuracy can be achieved with unsophisticated devices.



R. LIEBER received his BSME from the University of Miami and his MS from Drexel Institute of Technology. At the Army's Frankford Arsenal from 1950 to 1952, he was responsible for hydraulic circuit development. Joining RCA in 1952, Mr. Lieber contributed to development of the AN/APN-42 subminiaturized pulse radar altimeter and the "Black Cat" low-altitude penetration system. He then became responsible for the integration of the MIT-SPIRE System with the RCA-developed K-band terrain clearance radar. From 1956 to 1958 he undertook functional engineering of inertial navigation systems and self-stabilized missile-seeker systems. Since 1958, Mr. Lieber, as Leader of an Analysis group, is responsible for major portions of DAMP and advanced tracking and guidance systems.

DR. SIDNEY SHUCKER received the BSEE in 1948 from Drexel Institute, and the M.S. in EE in 1949 and the Ph.D. in 1961 from the Univeristy of Pennsylvania. From 1949 to 1951, he worked on development of radar circuits and instrument servos. Since joining RCA in 1951, he has worked in the fields of servomechanisms and systems engineering.